Simulations of beam optics and bremsstrahlung for high intensity and brightness channeling radiation

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ABSTRACT

This paper presents X-ray spectra of channeling radiation expected at the FAST (Fermi Accelerator Science and Technology) facility in Fermilab. Our purpose is to produce high brightness quasi-monochromatic high energy X-rays using a low energy electron beam below 100 MeV [2–4]. As an example, CR from a 14.6 MeV energy electron beam has been shown to emit 16 keV X-rays [5]. By comparison synchrotron radiation, currently the main X-ray source, requires a few GeV electron beams to generate X-rays of 16 keV. Moreover, intensities of CR are higher than those of parametric X-ray radiation and transition radiation which are also produced using a crystal and an electron beam.

Our purpose is to produce high brightness X-rays using a low-emittance electron beam, and to demonstrate that CR can be used as a compact high-brightness X-rays source. We plan to conduct CR generation experiments at the FAST (Fermi Accelerator Science and Technology) facility in Fermilab. Low emittance electron beams can be generated at the FAST injector which consists of a CsTe photocathode located in a 1+1/2-cell RF gun followed by two L-band (1.3 GHz) superconducting accelerating structures [6,7]. The electron energy can reach up to ~50 MeV downstream of the last superconducting cavity. A diamond single crystal will be used because of its low atomic number Z, high Debye temperature, and large thermal conductivity. The crystal is oriented so that the electron beam propagates parallel to the (110) plane of the crystal. The expected spectra of CR at FAST were reported earlier [8,9]. In this report, we present detailed calculations of the beam optics, background to CR from bremsstrahlung, electron beam distributions after the diamond crystal with and without channeling, and the X-ray detector system for the CR experiments.

In Section 2 of this paper, the energy, yield, and brilliance of CR are described. In Section 3, the FAST photoinjector is shown, where the details of the superconducting cavities, magnets and expected emittances are described. The beam optics and the X-ray spectra including bremsstrahlung background for different charges are shown in Sections 4 and 5. The electron distributions after going through the crystal are shown in Section 6, an X-ray detector system using Compton scattering to avoid pile-up for high charge operations is described in Section 7 and conclusions are presented in Section 8.

1. Introduction

Channeling radiation (CR) can be generated when charged particles such as electrons and positrons pass through a single crystal parallel to a crystal plane or axis [1]. The main advantage of CR is to produce quasi-monochromatic high energy X-rays using a low energy electron beam below 100 MeV [2–4]. As an example, CR from a 14.6 MeV energy electron beam has been shown to emit 16 keV X-rays [5]. By comparison synchrotron radiation, currently the main X-ray source, requires a few GeV electron beams to generate X-rays of 16 keV. Moreover, intensities of CR are higher than those of parametric X-ray radiation and transition radiation which are also produced using a crystal and an electron beam.

Our purpose is to produce high brightness X-rays using a low-emittance electron beam, and to demonstrate that CR can be used as a compact high-brightness X-rays source. We plan to conduct CR generation experiments at the FAST (Fermi Accelerator Science and Technology) facility in Fermilab. Low emittance electron beams can be generated at the FAST injector which consists of a CsTe photocathode located in a 1+1/2-cell RF gun followed by two L-band (1.3 GHz) superconducting accelerating structures [6,7]. The electron energy can reach up to ~50 MeV downstream of the last superconducting cavity. A diamond single crystal will be used because of its low atomic number Z, high Debye temperature, and large thermal conductivity. The crystal is oriented so that the electron beam propagates parallel to the (110) plane of the crystal. The expected spectra of CR at FAST were reported earlier [8,9]. In this report, we present detailed calculations of the beam optics, background to CR from bremsstrahlung, electron beam distributions after the diamond crystal with and without channeling, and the X-ray detector system for the CR experiments.

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2. Channeling radiation

This section briefly presents the energy, yield, and brilliance of CR. When an electron beam travels with a sufficiently small transverse momentum with respect to a crystal plane, the electrons can be captured in bound states of the crystal's transverse potential, and consequently emit CR. Electron motion in the crystal is similar to that in an undulator, and the vibration period is very short, therefore, high energy X-rays

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can be emitted using a comparatively low energy electron beam. An important requirement for CR is that the electron beam divergence at the crystal must be smaller than the critical angle $\theta_c$ [10]. If beam divergences are larger than the critical angle, electrons in the crystal have too large a transverse momentum to be captured by the crystal potential. The critical angle $\theta_c$ for CR is given by

$$
\theta_c = \sqrt{\frac{2V_{\text{max}}}{mc^2(\gamma^2 - 1)}} \approx \sqrt{\frac{2V_{\text{max}}}{mc^2\gamma^2}}.
$$

(1)

where $V_{\text{max}}$ is the transverse potential of a crystal, $m$ is the electron mass, $c$ is the speed of light, and $\gamma$ is the Lorentz factor. For an electron energy of 43 MeV and a diamond crystal with the (110) plane, the critical angle $\theta_c = 1.1$ mrad using Eq. (1).

The CR energy spectrum for electron beams below 100 MeV, derived by solving Schrödinger's equation, is discrete [2–5]. Electrons bound by the potential of a crystal plane or axis have discrete energy levels, and spontaneous transitions between the energy states generate quasi-monochromatic CR. On the other hand, for electron energies above ~100 MeV, the energy spectrum of CR can be described by classical electrodynamics and it is broad and continuous [10]. The CR energies for electron energies below 100 MeV are given by [2,5]

$$
E_y = 2y^2 \left[ \left( \frac{\varepsilon_i - \varepsilon_f}{1 + \gamma^2 \theta_c^2} \right) \right],
$$

(2)

where $\theta$ is the angle of the emitted photon from the incident electron, $\varepsilon_i$ and $\varepsilon_f$ are the energy eigenvalues of electrons in energy levels $i$ and $f$. This equation also shows that the CR energy can be tuned by changing the electron energy since $E_y \propto \gamma^2$. The most intense spectral lines for 43 MeV electrons going through the diamond and for photons emitted along the direction of the incident electron beam are as follows: 67.5 keV (transitions from the excited state [2] to excited state [1]), 110 keV (transition from state [1] → [0]) and 51 keV (transition from [3] → [2]). The spectrum is shown in Fig. 8.

The photon yield from state $|i\rangle \rightarrow |f\rangle$ can be found in [3,9,31] and explicitly depends on the occupation probability of the channeling state. This probability is essentially determined by the overlap integral of the electron eigenfunction in that state and a plane wave state. A low beam divergence increases the occupation probability of the excited states which in turn increases the photon yield from those states.

In general, the quality of X-ray sources such as synchrotron radiation and XRF is evaluated with the spectral brilliance (photons/s/mm²/mrad²/0.1%BW). The average brilliance of CR emitted from electrons can be expressed as [9]

$$
B_{\text{ave}} \left[ \text{photons/s/mm²/mrad²/0.1%BW} \right] = \frac{I_{\gamma} \gamma Y(\sigma_f^2)^{0.3} \varepsilon_i \Delta E_z/E_r}{\varepsilon_i \Delta E_z/E_r \cdot E_r 
\text{erf} \left[ \frac{\theta_c}{\sqrt{2} \sigma_f} \right],}
$$

(3)

where $I_{\gamma}$ is the average electron beam current, $\varepsilon$ is the elementary electron charge, $Y$ is the total photon yield per electron, $\sigma_f$ is the normalized emittance, $\theta_c$ is the critical angle, see Eq. (1), $\Delta E_z/E_r$ is the relative width of the X-ray line, $\sigma_f$ is the electron beam divergence, and $\text{erf}$ is the error function. According to Eq. (3), the average brilliance is proportional to $1/\sigma_f^2$, which shows that beam sizes at a crystal location should be small to generate high brightness CR.

3. FAST photoinjector

In this section, the FAST photoinjector and parameters of the electron beam for CR experiments are described. The main components in the beamline are a Cs$_2$Te photocathode RF gun, two superconducting accelerating structures with TESLA style 9-cell cavities, quadrupole magnets, a chicane, a vertical bending magnet, and a beam dump [6]. Fig. 1 shows the layout of the photoinjector. The RF gun consists of a cathode with a molybdenum disk coated with Cs$_2$Te mounted on the back plate of a 1+1/2-cell normal-conducting cavity operating at 1.3 GHz with a repetition of 5 Hz. The RF gun is identical to the one developed for the FLASH facility at DESY [7]. A bunch train repeated at 3 MHz with 1-ns duration is produced by irradiating the cathode with an ultraviolet laser pulse (wavelength of 263 nm). The electrons have an energy of ~5 MeV at the exit of the RF gun, and are accelerated up to 150 MeV in the range 45–50 MeV in the two superconducting structures operated at an RF frequency of 1.3 GHz.

The goniometer housing the crystal and the X-ray detector for the CR experiment are located at 17 m and 18.5 m, respectively, from the photocathode. The goniometer stage can be rotated around vertical and horizontal axes for an incident electron beam direction and can slide horizontally. The movable target holder houses: (1) a clear aperture, (2) the diamond crystal and (3) a 50-micron thick Al foil. The hole is used when the crystal is not needed. Intercepting the beam with the foil generates bremsstrahlung that can be used to calibrate the detection system. The bremsstrahlung also provides a coarse calibration signal to center the beam on the foil and indirectly on the crystal as the foil and crystal are inserted using a calibrated stepping motor.

This dipole magnet after the goniometer kicks the beam vertically by 22.5° towards the beam dump. The quadrupole magnets used to control electron beam sizes have a maximum gradient of 6.6 T/m at an energy of 50 MeV, their bore diameter is 54.6 mm and the effective magnetic length is 617 mm. Also, eight steering magnets are inserted in the beamline to correct the electron beam trajectory. Each steering magnet is capable of a maximum kick of 7.5 mrad to a 50 MeV electron beam.

The chicane displayed in Fig. 1 is commonly used for bunch compression and energy collimation. The four dipole magnets in the chicane provide bending angles of $(+\gamma, -\gamma, +\gamma, -\gamma)$ respectively yielding a longitudinal dispersion of $R_{x,y}$. $R_{x,y}$ is the matrix element that connects the path difference to the energy deviation and is given by $R_{x,y} = \int_{-\infty}^{+\infty} ds \cdot -0.18$ m, where $s$ is the bend radius of the dipole magnet. In the CR experiment, the main purpose of using the chicane is to collimate away dark current of lower energy than the main photocathode current.

Electron beam dynamics from the photocathode to downstream of the second superconducting cavity (8 m) were simulated, including space charge effects, using the tracking program ASTRA [12] for bunch charges ranging from 1 pC to 3.2 nC. Table 1 shows the twiss parameters, the normalized emittances, and the energy spreads for bunch charges of 1, 20, and 200 pC. The rms bunch length is about 3 ps.

<table>
<thead>
<tr>
<th>Charge (pC)</th>
<th>$\sigma_x (\text{mm-rad})$</th>
<th>$\sigma_y (\text{mm-rad})$</th>
<th>$\Delta E/E$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.02</td>
<td>~39.3</td>
<td>0.1</td>
</tr>
<tr>
<td>20</td>
<td>0.19</td>
<td>~3.8</td>
<td>21.3</td>
</tr>
<tr>
<td>200</td>
<td>0.52</td>
<td>~3.6</td>
<td>18.9</td>
</tr>
</tbody>
</table>

**Table 1**

Twiss parameters, normalized emittances, and energy spreads for 1, 20, and 200 pC at 8 m from the photocathode, from ASTRA simulations.

<table>
<thead>
<tr>
<th>Charge (pC)</th>
<th>$\sigma_x (\text{mm-rad})$</th>
<th>$\sigma_y (\text{mm-rad})$</th>
<th>$\Delta E/E$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>1.3</td>
<td>(1.3, 1.3)</td>
</tr>
<tr>
<td>20</td>
<td>3</td>
<td>4.1</td>
<td>(4.1, 4.0)</td>
</tr>
<tr>
<td>200</td>
<td>5</td>
<td>9.6</td>
<td>(9.6, 9.6)</td>
</tr>
</tbody>
</table>

**Table 2**

Twiss functions, minimum beam sizes, and beam divergences at the crystal for different charges. Initial conditions are shown in Table 1.

4. Beam optics solutions for CR

In this section, we present optics solutions for three values of the bunch charge and for two scenarios: (1) high brightness solutions with low beam sizes and divergences close to the critical angle (consistent with the beam emittance) and (2) high yield solutions with a large beam size and low divergences at the crystal.
4.1. Beam optics solutions for high brightness CR

For bunch charges of 1, 20, and 200 pC, the beam optics from a point 8 m downstream of the photocathode which is just after CC2 (the second superconducting cavity), to the beam dump was simulated with initial parameters in Table 1 using SAD (Strategic Accelerator Design) computer code [13]. Fig. 2 shows the beta functions obtained by minimizing the beam size at the crystal for a bunch charge of 20 pC. The beam optics simulated for 1 pC and 200 pC shows similar behavior. The optics matching was done with two sets of triplet magnets. Their polarities are (+, −, +) and (−, +, −); “+” and “−” mean horizontal and vertical focusing, respectively. The quadrupole magnets used in matching are constrained to a maximum gradient of 6 T/m. Starting with the initial conditions in Table 1, the minimum beta functions at the crystal, and the beam sizes and the beam divergences at the crystal for three charges are shown in Table 2. The beam sizes are largest in the final quadrupole magnets before the crystal, however, the beam sizes are much smaller than the radius of the beam pipe.

The calculated beam sizes ($\sigma_{x,y} = \sqrt{\sigma_{x,y}\sigma_{x,y}}$) for minimum beta functions are smaller than those obtained by particle tracking which includes chromatic aberrations in the quadrupoles. Fig. 3 shows beam sizes at the crystal as a function of the beta functions for the different charges calculated with both methods. Assuming sextupole magnets cannot be placed in the channe (the only region of non-zero dispersion) to correct the chromaticity, the only other option to keep chromatic effects small is that the electron beams have a small enough momentum spread. Fig. 4 shows the dependence on momentum spread of the beam sizes at the crystal simulated by particle tracking when the momentum beta function is 3 mm at the crystal. The momentum spread affects the horizontal size more due to strong horizontal focusing in the last triplet magnet. Also, this shows that the momentum spread should be less than 0.1% at the crystal. The energy spreads expected for (1, 20) pC, shown in Table 1, are about 0.1% but larger for 200 pC. Therefore, the effect of the chromatic aberration should not be significant for the two lower charges.

The beam divergences at the crystal for 1, 20, and, 200 pC are 0.3, 0.9, and 1.1 mrad, respectively. For the different charges, 94% (1 pC), 50% (20 pC), and 34% (200 pC) of particles in a bunch satisfy the channeling condition of the critical angle, while the other particles pass through without channeling. Although the brilliance decreases slightly at higher charge, the loss of channeled particles has little overall effect on the brilliance because it depends on the square of the beam size at the crystal.

4.2. Beam optics solutions for high yield CR

To produce high yield CR, the beam divergence at the crystal should be much smaller than the critical angle. This means that beam size at the crystal will be large due to the conservation of beam emittance and strong focusing is not required. Hence the chromatic aberration can be ignored (see Fig. 3). We simulated the beam optics so that the electron beam has divergences of 0.1 mrad with an optics waists (twiss parameter $\alpha_{x,y} = 0$) at the crystal for each charge. The beta functions along the beamline are shown in Fig. 5 when the bunch charge is 20 pC. The beam optics simulated for each charge is different due to different initial twiss parameters but shows similar behavior. For charges of 1 pC, 20 pC, and 200 pC, the beam sizes at the crystal are 2.9 μm, 22 μm, and 61 μm, respectively with a constant beam divergence of 0.1 mrad. The beam sizes obtained from analytical calculation of the betatron beam size without chromatic effects are consistent with those from particle tracking.

5. Background from bremsstrahlung

Bremsstrahlung (BS) X-rays are produced when relativistic electrons passing through a crystal are scattered by the atomic nuclei. The
spectrum of BS is continuous, covering a wide energy range from microwave to hard X-rays. The maximum BS energy can extend to nearly the electron energy. BS is undesirable background in CR experiments since it is emitted in the same forward direction as CR. In order to examine the effect of BS in our experiment, we estimated the photon count and the energy distribution of the background registered in the detector with Geant4 [14] simulations. The X-ray detector with an aperture of $3 \times 3 \text{ mm}^2$ is assumed to be located at 1.5 m from the target crystal so that the detector's acceptance is 2 mrad. The layout showing the X-ray detector and simulation geometry is shown in Fig. 1. In the simulations, $10^6$ particles (20 pC/bunch) with an energy of 43 MeV and a diamond crystal with a thickness of 168 μm are used.

**Fig. 4.** Beam size distributions at the crystal for different momentum spreads for 20 pC when the beta functions at the crystal are 3 mm. Left: horizontal plane, right: vertical plane.

**Fig. 5.** Beta functions for the low divergence solution along the transport line from the last cavity to the beam dump when the bunch charge is 20 pC. The beam divergence at the crystal is matched to 0.1 mrad. Colors of the lines and boxes have the same meaning as in Fig. 2.

**Fig. 6.** Angular and energy distributions of photons emitted in the detector with an acceptance of 2 mrad. Left: Angular distribution of bremsstrahlung photons in the detector. Right: Bremsstrahlung energy spectrum in the detector.

**Fig. 7.** Number of photons hitting the detector depending on beam divergences and beam sizes of incident electrons.

Fig. 6 shows the scattering angle and the energy distribution of BS photons in the detector. About $10^4$ photons enter the detector, and the photon intensity decreases below 10 keV due to self-absorption in the Al foil and then becomes zero below 3.9 keV due to the production threshold in Geant4. Also, the photon count hitting the detector as a function of beam size for different beam divergences at the crystal is plotted in Fig. 7. For the electron beam size below 0.1 mm at the crystal, the number of photons going into the detector is substantially constant at $\sim 10^3$ particles. However, it decreases when the electron beam size is over 0.1 mm. For an electron bunch charge of 20 pC, we expect about $10^4$ background BS photons per electron bunch will be registered in the detector, since the beam size at the crystal is desired to be below 0.1 mm. Also, the ratio of background photons in the detector to incident electrons is approximately $10^{-4}$, therefore, about $5\times10^2$ and $10^3$ photons
per bunch for bunch charges of 1 pC and 200 pC respectively will be registered as background.

The expected channeling spectra, including the background, when a 43 MeV electron beam is incident on a 168 μm thick diamond crystal parallel to the (110) planes, are shown in Fig. 8. These CR spectrum calculations without the background were done using a Mathematica notebook which was significantly corrected and modified from the published version [13]; this modified version was used to successfully compare simulations [9] with experimental results from a previous channeling experiment [5]. The processes of de-channeling and re-channeling in a crystal are taken into account, which affects populations in bound states and reduces photon yields. The CR photon count can be calculated using a free parameter \( n_f \), which is roughly the number of excited bound states contributing to the photon yield. The appropriate \( n_f \) was determined by comparing simulations with the previous experiment [9]. Complete details of the model with this parameter \( n_f \) including the modified equations for the propagation of the electron probability distribution along the crystal can be found in [5]. The "CR high" label in Fig. 8 corresponds to the result for a case with very little dechanneling \( n_f = 2 \), "CR mid" is the result for the case \( n_f = 17 \) estimated to be the best fit with the experimental values, and "CR low" is a more conservative result with \( n_f = 13 \) corresponding to a lower yield. The X-rays with discrete energies of 110 keV (transition: [1] \( \rightarrow [0] \)), 67.5 keV (transition: [2] \( \rightarrow [1] \)) and 51 keV (transition: [3] \( \rightarrow [2] \)) are generated along the electron direction. The ratios of CR signal for "CR mid" \( n_f = 17 \) to the BS background are about 8 at 110 keV, 7 at 67.5 keV, and 4 at 51 keV. These theoretical values of signal to backgrounds imply that the CR signal should be clearly observable at the higher energy CR spectral lines.

We compared the brilliance and the number of photons of CR with those of synchrotron radiation (SR) at APS (Advanced Photon Source, Argonne, USA). Table 3 shows the number of photons per electron and the brilliance per electron of CR and SR. Those of CR are computed with "CR mid" in Fig. 8 and Eq. (3), and those of SR at APS were converted using values referred from [15]. Both the yield and brilliance are calculated per electron in order to remove the dependence on bunch charge. The photon yield per electron at a chosen energy is about a factor of 50 lower with CR compared to SR, even though the electron beam energy is about 160 times smaller in the linac for CR compared to the storage ring for SR. However, the brilliance per electron is more than 8 orders of magnitude higher for SR compared to CR, mainly due to the higher energy and lower vertical emittance in the storage ring. The advantage of CR is that it can produce quasi-monochromatic hard X-rays with energy tunability using a cheaper accelerator with a compact footprint.

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Comparison of CR at FAST and synchrotron radiation at APS.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channeling radiation</td>
<td>Synchrotron radiation</td>
</tr>
<tr>
<td>Electron beam energy</td>
<td>43 MeV</td>
</tr>
<tr>
<td>Bunch charge</td>
<td>20 pC</td>
</tr>
<tr>
<td>Beam emittance ((\varepsilon_{x,y}))</td>
<td>(2.5, 2.3) nm-mrad</td>
</tr>
<tr>
<td>Photons/electron (*1\times10^7) @67.5 keV</td>
<td>(5\times10^7) @20 keV</td>
</tr>
<tr>
<td>Brightness/electron</td>
<td>(1.5\times10^7) @67.5 keV</td>
</tr>
</tbody>
</table>

\(5\times10^7\) @20 keV

6. Electron beam distributions after crystal

This section describes electron beam sizes and beam divergences after the electron beam passes through the diamond crystal with and without channeling. This beam experiences multiple scattering events with atomic nuclei, and the beam divergence grows after passing through the crystal. Therefore, the electron beam emittance increases, which could cause particle loss from scraping at the beam pipe. The multiple scattering depends on whether the beam is channelled or not in the crystal. The rms scattering angle \( \theta \) for an electron that is not channelled depends on the crystal thickness \( L \), its atomic number \( Z \), its mass number \( A \), and the electron momentum \( p \), as [16,17]

\[ \theta = 13.6(\text{MeV}) \sqrt{\frac{L}{L_{\text{rad}}}} \left(1 + \frac{0.038n}{L/L_{\text{rad}}} \right) \]

\[ L_{\text{rad}} = \frac{716.4(\text{eV}/\text{cm}^2)^{1/4}}{Z(Z+1)(0.287/\sqrt{Z})} \]  

where \( \text{eV} \) is the velocity of the electron, \( L_{\text{rad}} \) the radiation length of the crystal. The rms scattering angle after an electron of 43 MeV passes through a 168 μm thick diamond crystal can be computed to be about 8.3 mrad. However, the beam divergence for a channelled electron has been estimated to be smaller than that obtained from Eq. (4) [18,19]. Using recently added modules in Geant4 [14,20], we estimated the scattering angles and energy spreads of the electron beam passing through the crystal under both channeling and non-channeling conditions.

The angular scattering of the electron after the crystal calculated using Geant4 is shown in Fig. 9. The initial beam divergence at the crystal is set to be 0.1 mrad, and \(10^7\) electrons are used in both cases. The left plot in Fig. 9 shows the initial angular distribution of electrons at the crystal entrance and the right plot shows the distribution after the crystal with and without channeling. The standard deviation of the entire distribution (including the tails) without channeling is about 10.1 mrad, about 100 times larger than that before the crystal. On the other hand, the standard deviation of the distribution after channeling is smaller than 7.7 mrad, or approximately 0.8 times the value without channeling. Fig. 10 shows the beam sizes of electrons at a monitor in front of the detector, with and without channeling in the crystal. The beam monitor is assumed to be at 1.5 m from the crystal. Without channeling in the crystal, the beam sizes at the monitor are \((X,Y) = (15.4 \text{ mm}, 15.2 \text{ mm})\) while with channeling, they decrease to \((11.3 \text{ mm}, 11.1 \text{ mm})\).

When relativistic electrons pass through the crystal, the energy loss is caused both by ionization losses and radiation losses (bremsstrahlung). In our CR experiments, the ionization loss dominates the energy loss, because the electron energy of 43 MeV is lower than the critical energy in diamond \(E_c = 84 \text{ MeV}\) [16]. The ionization loss can be analytically calculated by the Bethe-Bloch equation [16], and the mean energy loss for our set of parameters is about 100 keV, or \(\sim 0.2\%\) of the initial electron energy. Fig. 11 shows the electron energy distributions after the crystal, calculated with Geant4. The green curve in Fig. 11 is the initial energy distribution of electrons with an energy spread of 0.1%, the blue curve is the energy distribution after the crystal without channeling, and the red curve is after channeling. For the case without channeling, the energy distribution has a peak at 42.9 MeV and the energy loss is about 100 keV, in agreement with the analytical calculation. On the other hand, for the case with channeling, the peak is at 42.95 MeV and the energy loss is about 50 keV.
Fig. 9. Angular distributions of a 43 MeV electron beam before and after going through the diamond crystal with the thickness of 168 μm. Left plot [mrad scale]: the distribution at the crystal entrance. Right plot [rad scale]: the beam distribution with and without channeling. Standard deviations with and without channeling are 7.7 mrad and 10.1 mrad, respectively.

Fig. 10. Beam positions and sizes (x, y) after the non-channeled (left) and the channeled (right) electron beam at 1.5 m after the crystal.

Fig. 11. Energy distributions of the electron beam before and after the diamond crystal. The green curve shows the initial energy distribution. The red and blue curves are the distributions with channeling and without channeling, respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

7. Compton scattering for X-ray detector

The experimental layout in Fig. 1 shows an X-ray detector downstream of the crystal. This configuration will be used at low bunch charges when the photon rate emitted into the detector’s acceptance is about 1 photon per bunch. At higher bunch charge and the nominal laser frequency of 3 MHz, the photon rate will be high enough to cause photon pile up in this detector. Two or more photons arriving within the detector’s response time (about 25 μs) will be registered as a single photon with an energy which is the sum of all the photon energies leading to a wrong spectrum. In this section, we discuss the use of a second X-ray detector placed orthogonal to the beamline which will detect photons Compton scattered from a plastic plate to avoid this pile up effect.

In Section 5 we had estimated that the number of BS photons for a bunch charge of 1 pC entering the detector would be about 500 photons per bunch or $7.5 \times 10^6$ photons/s for a pulse repetition rate of 5 Hz. Since this does not include the CR photons, the pile up will be considerable even at low bunch charge.

For low photon energies, the photon flux into the detector can be reduced using an attenuator such as Al and Brass, and we can extract the true photon rate by calculating the number of photons absorbed into the attenuator with the absorption data supplied by NIST [21]. By contrast, for high photon energies, such attenuators are not effective due to low absorption cross sections. This will be true in our experiment with expected CR photon energies ranging from 50 keV to 110 keV. Thus, we will utilize Compton scattering that can significantly reduce the photon rate for high photon energies.
Compton scattering results from the interaction of a photon with free electrons in a material substance. The scattered photons experience energy loss, resulting in shifts to longer wavelength. The differential cross section for Compton scattering is given by the Klein–Nishina formula:

$$\frac{d\sigma}{d\Omega} = \frac{r_e^2}{2} \frac{1}{(1 + h\nu(1 - \cos\theta))^2} \left( 1 + \cos^2\theta + \frac{(h\nu_p)^2(1 - \cos^2\theta)}{1 + h\nu(1 - \cos\theta)} \right),$$

(6)

$$h\nu = \frac{1}{1 + \frac{r_e^2}{m_e c^2}(1 - \cos\theta)},$$

(7)

where $r_e$ is the classical radius of the electron, $h\nu_p$ is the energy of the incident photons, and $\theta$ is the scattering angle. Eq. (6) shows that the photon’s differential cross section has a minimum at 90° and most photons are scattered in the forward and backward direction to the incident photons. Therefore, the detector should be placed orthogonal to the beamline (incident photons) and the layout of the second X-ray detector is shown in Fig. 12. Also, using Eq. (7), X-ray energies of 110 keV, 70 keV, and 50 keV scattered at 90° can be estimated to have their energies shifted to 90 keV, 60 keV, and 45 keV, respectively.

Since photons with energies of 110 keV, 70 keV, and 50 keV are expected in our experiment, the target material should have an atomic number below 30 owing to the dominance of Compton scattering in this energy range, from the three major types of photon interactions (photoelectric effect, Compton scattering, pair production) with matter [22].

In order to select appropriate materials to use in our experiment, we have tried diamond, Al, PMMA (Poly(methyl methacrylate), and PVC (Polyvinyl chloride) plates, each with a thickness of 2 mm. Photon counts in the X-ray detector were simulated using Geant4. As an initial condition, $10^6$ photons with an energy of 70 keV, and an energy spread of 10% was used. The detector with the aperture of $3 \times 3$ mm$^2$ is assumed to be located at 0.375 m from the Compton scattering plate, see Fig. 12.

Fig. 13 shows the number of Compton scattered photons and their energies hitting the detector for the different materials. Photons with energies of ~60 keV and energy spreads of ~10% are scattered in the detector; the simulated energies agree with the analytical calculation with Eq. (7). The Compton scattered photon count decreases with an increase of the atomic number $Z$ due to the absorption within the material. For the organic materials, the number of photons in the detector is about 200 and decreases by seven orders of magnitude, therefore, we decided to use PVC or PMMA as the material for Compton scattering.

8. Conclusions

The topics discussed include: (1) beam optics for both a high brightness and high yield solutions, (2) CR photon yields including background from bremsstrahlung (BS), (3) electron beam distributions with and without channeling through a diamond crystal, and (4) an X-ray detector system to avoid pile up.

The beam optics for the two operations requires different solutions: low beam sizes for high brightness X-rays and low beam divergences for high yield X-rays. We simulated optics solutions for three bunch charges of 1 pC, 20 pC, and 200 pC with the SAD code. Small beam sizes require strong focusing and the impact of chromatic aberrations become more significant as the bunch charge increases due to a corresponding increase in the momentum spread. Thus, for the high brightness solution, the minimum rms beam sizes at the diamond crystal were (1, 4, 10) μm respectively for the three bunch charges studied. As for the high yield version, we obtained beam optics solutions with a beam divergence of 0.1 mrad at the crystal for the different bunch charges. This divergence is much less than the critical angle for channeling, about 1 mrad at 43 keV.

To examine the effect of the BS background on the CR signal, we simulated the BS generated from electrons propagating in the diamond crystal using Geant4. The signal to background ratio varies from 4 at the 50 keV CR spectrum line to 8 at the 110 keV CR line.

Electron beam distributions after propagating through the crystal with and without channeling were simulated with Geant4. The beam divergence after channeling was 0.8 times smaller than the case without channeling. Consequently, under channeling conditions, the electron beam size after the crystal was smaller in both planes. These results suggest that a smaller emittance growth and beam size after the crystal could be used to test for channeling in the initial stage of the CR experiments.

To avoid pile up and saturation of the X-ray detectors, we utilized a second X-ray detector orthogonal to the beam line and Compton scattering to reduce the photon count. Geant4 simulations show that a 2-mm thick scattering plate made of organic material such as PVC or PMMA can reduce the photon number in this detector by seven orders of magnitude.

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