In this paper we question the pioneering work of Todaro, which states that rural-to-urban labor migration in less developed countries (LDCs) is an individual response to a higher urban expected income. We demonstrate that rural-to-urban labor migration is perfectly rational even if urban expected income is lower than rural income. We achieve this under a set of fairly stringent conditions: an individual decision-making entity, a one-period planning horizon, and global risk aversion. We obtain the result that a small chance of reaping a high reward is sufficient to trigger rural-to-urban labor migration.

I

That poor people in less developed countries (LDCs) undertake migration and, in particular, rural-to-urban migration as an act of rational choice has long been a major theme in development economics research. There is also wide recognition that, at least in some cases, migration constitutes an actuarially unfair risk with income earning in the urban sector often not being guaranteed prior to the migrants’ arrival there.
When subjected to empirical testing, the popular expected income hypothesis of rural-to-urban migration (associated most closely with Todaro [1969]) does not fare well in terms of either the sign of the coefficients or their statistical significance. In addition, in many cases the expected income in the urban area is not larger than the expected income in the rural area. Combined, these observations point to an empirical paradox: how is it that calculative behavior by rational people results in choice of an actuarially unfair risky prospect? In this paper we offer two explanations but first list and briefly discuss four additional responses to this question. We reject the first two; the third and fourth have been proposed elsewhere (see Stark and Levhari 1982) and require the acceptance of special assumptions.

One possible response to the problem posed above is that rural-to-urban migrants are risk loving. Such a characterization draws particularly on the evidence that migrants are usually selectively drawn from the rural areas with respect to characteristics that are closely correlated with love of risks: young age, low ratio of location-specific to location-transferable human capital, and so forth (Kuznets 1964; Myrdal 1968; Sahota 1968). Thus an argument can be made that a sample-selection problem might creep in: only the risk-preferring leave, so the remaining people are risk averse. What induces us to reject this explanation is the evidence that in other domains of economic life farm people in LDCs (including the ones who subsequently turn out to be migrants) ordinarily shirk risks (Schultz 1964; Roumasset 1976).

A second explanation is bounded rationality: human rationality is limited and bounded by the situation and by human computational powers (Simon 1983). People can deal with only one major problem at a time. People may therefore not be able to deal with all the possible implications of their migration decision. What makes us uncomfortable about this explanation is that bounded rationality seems to be a useful explanation in situations in which the world is mostly intersectionally "empty" in the sense that most variables are only weakly related to other variables. Then the world may be factorable into separate problems. But such factorability does not seem to apply when a different risk is inherent in every alternative of income earning.

A third response is that rural-to-urban migration is perfectly consistent with a person’s aversion toward risk even if the risks associated with urban income earnings are initially high. Clearly, this line of reasoning introduces an intertemporal utility function. It seems that risks (variability) associated with urban employment diminish with time and, after some

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1 Good examples of recent studies that reach this conclusion are Banerjee and Kanbur’s (1981) study for India, Salvatore’s (1981) study of migration from the rural south to the urban north in Italy, and Garrison’s (1982) study for Mexico.
initial high-risk period, may be relatively low, that is, lower than the
typical risk associated with agricultural production. A person will
therefore engage in rural-to-urban migration if he attaches a premium
to the early resolution of (much of) his lifelong risks. He trades in
"medium-level risks" for immediate higher, but subsequent lower, risks.
This is, of course, likely to be particularly relevant if the individual is
young, so that the low-risk period he faces after migrating is especially
long. However, accepting this explanation imposes narrow bounds on
the time discount factor.

A fourth response rests on broadening the decision-making entity
perspective rather than deepening the time perspective. Although the
modal rural-to-urban migrating unit in LDCs is an individual, there are
strong theoretical and empirical reasons to suggest that the decision­
making entity is often the family, of which the individual is a member.
Migration by a family member is then warranted when it facilitates
reduction in total familial risk via diversification of earning sources. For
the migration strategy to make sense in this context, one of the following
two conditions is required: that, for example, the head of the family
keeps perfect control over the migrant or that a cooperative arrangement
is struck between the two decision makers—the family and the migrant—
involving intrafamilial trade in risks, coinsurance arrangements, devices
to handle principal agent problems, moral hazard problems (the migrant
understates his success in the urban area, or the migrant increases his
standard of living appreciably, thus producing a smaller surplus), and
contract enforcement problems (the migrant admits his success but
refuses to share it with his family) and, overall, striking a mutually
beneficial, intertemporal, self-enforcing contractual arrangement. If mi­
gration by two or more family members is allowed, these problems are
compounded as issues of coalition formation and Prisoner's Dilemma
creep in. (The latter situation might arise when every migrant member
wishes the family to enjoy remittances yet prefers that another migrant
member remit more and himself less.) This promising approach is
presented in related papers (Stark 1983; Lucas and Stark 1985).

However, we depart from this direction here for three specific
reasons. First, in many situations rural-to-urban migration in LDCs is an act of
individual choice. Second, we wish to relate our analysis to the pioneering
work of Todaro (1969), which is cast in an individualistic context. This
enables us to criticize the work constructively and fairly and to offer an
alternative theory drawing on the characteristic features of LDCs' capital
markets. Furthermore, it also enables us to explain temporary migration,
which Todaro's theory certainly does not. Third, while it is interesting
and relevant in some important contexts, the portfolio diversification
approach must be pursued with care; the delineation of an efficient
portfolio is highly complex and very sensitive to small changes in the
parameters of the distributions involved, namely, their means, their spreads, and their correlations. Consequently, the conclusion about whether all, some, or no family members will migrate is not that clear-cut. But the very question we are addressing requires an all-or-nothing approach. Might a risk-averse person migrate or not? Is migration consistent with global risk aversion? This paper attempts to provide a positive answer to both these questions.

Before turning to our analysis, we wish to address briefly one final point. The explanation of the rural-to-urban migration phenomenon has many policy implications. Suppose that in an LDC an institutional action aimed at reducing rural-to-urban migration is deemed desirable. Clearly, different sets of policies would be relevant, depending on what it is that fuels migration. For example, if migration is fueled by risk diversification resulting from the incompleteness of insurance markets, then it would be efficient to shift from exclusive attempts to narrow urban-to-rural wage differentials (which have not been very successful) toward the creation and/or perfection of insurance markets. If, as we shall suggest here, a main cause of rural-to-urban migration—a labor market phenomenon—is capital market imperfections, then policy to constrain migration should aim at enhancing access to, and improving the competitiveness of, capital markets.

II

As is well-known, an individual who is not everywhere risk averse may rationally engage in both insurance and actuarially unfair gambling. Thus, following the classic work of Friedman and Savage (1948), if the individual's utility function is in turn concave, convex, and concave, this might explain why under certain circumstances he may expose himself to some risks at the very same time that he pays to shield himself against other risks. The migration decision might then be explained by the existence of a Friedman and Savage utility function, where the migrant is on the convex part of the utility function.

Yet Friedman and Savage's explanation has been recognized as an essentially ad hoc formulation of the utility function created specifically to accommodate a bothersome economic phenomenon, namely, simultaneous gambling and insurance. In contrast, it has been suggested (see Appelbaum and Katz 1981) that, even if individuals are everywhere risk averse, they may still undertake actuarially unfair risks. A circumstance under which this is likely to occur is a situation in which the yield to investment is an increasing function of the amount of money invested, that is, in the presence of imperfect capital markets, a critical feature of LDCs.

Let us demonstrate how this approach applies in the context of the migration decision. An individual with an initial wealth of $A$ rupees is
faced with the decision to migrate when the parameters facing him are as follows: if he migrates, there is a probability $q$ that he will obtain a job in the city. In the event that he does obtain a job, the net reward will be $W$ rupees. Alternatively, he may find himself jobless with a probability $(1 - q)$, in which case his net reward will be $-C$ rupees (as he will be eating into his own wealth).\(^2\)

If, on the other hand, the individual stays at home, then, because of the institutions of perfect intrafamilial sharing and some interfamilial sharing, his net reward will be $X$ rupees with certainty. To make the analysis simple without losing generality, let $X = 0$.

Clearly, if the individual is everywhere risk averse, and the story ends here, then a sufficient condition for him to choose not to migrate is that

$$\left(1 - q\right)C \geq qW, \quad (1)$$

that is, if migration yields no more (and possibly less) than an actuarially fair return.

However, if migration is viewed in a broader context, the story does not end here. Specifically, a number of recent studies discuss the effects of uncertainty, imperfect information, and various transaction costs on capital markets and show that the result may be that capital markets are characterized by certain imperfections. These studies establish both strong theoretical reasons (see Jaffe and Modigliani 1976; Barro 1976; Benjamin 1978; Braverman and Stiglitz 1982) and empirical evidence (see Eckstein 1961; Nerlove 1968) for believing that capital market imperfections of one kind or another are very likely to emerge when informational imperfections, high transaction costs, and so forth are present.

Clearly, within the context of the rural areas of LDCs, the imperfections described above are likely to be of even greater importance than within a developed economy; information in LDCs is, of course, very sparse and costly, given the lack of a modern communication infrastructure. Further, and perhaps for the same reason, several asset markets that exist in modern economies are completely absent in LDCs. This particularly applies to futures markets. The implication of the incompleteness of these markets is that a person will typically be unable to realize, on the current set of markets, the full potential value of his future wealth.

For example, a potential migrant might be able to obtain a loan locally, but this could require him to provide unpaid labor services to the money lender (cum landlord) at peak periods (thereby lowering his social status) and to accept a collateral valuation (e.g., for land or draught animals) below that given by the relevant local asset market (thereby devaluing his initial wealth). In such a case, migration may serve to

\(^2\) We rule out unemployment insurance payments from public agencies.
evade the institutional complexities and asymmetric bargaining power that characterize the incipient fragmented capital markets in LDC villages. These considerations, and the fact that at an early stage of the development process the return to physical investments may be extremely high, suggest that, at least for a certain range of wealth (that which, e.g., enables a farmer to adopt modern techniques), the rate of return on assets, $R$, is an increasing function of the level of investment, $Y$. Hence, we may write

$$R = R(Y),$$

(2)

where $R'(Y) > 0$ for at least some range of $Y$.³

Including these considerations in the individual's migration decision, an individual whose initial level of wealth is $A$ will be indifferent between migrating and not migrating if

$$U\{A[1 + R(A)]\} = qU\{(A + W)[1 + R(A + W)]\}$$
$$+ (1 - q)U\{(A - C)[1 + R(A - C)]\},$$

(3)

where $U$ is the Von Neumann–Morgenstern utility function defined on final wealth.

This indifference (isoutility) curve can be written as

$$C = G(W, A).$$

(4)

As is well-known, a necessary and sufficient condition for an individual with initial wealth $A$ not to engage in a fair bet is that the set of acceptable gambles be convex, implying that the boundary set as defined by $G(W, A)$ be concave.

Consider, therefore, the shape of the function $G$. As is proved in the Appendix, locally, at $W = C = 0$,

$$\frac{dC}{dW} = \frac{q}{1 - q} > 0$$

(5)

and

$$\frac{d^2C}{dW^2} = \frac{q}{(1 - q)^2} \left[ \frac{U''(\cdot)}{U'(\cdot)} + \frac{2R' + AR''}{(1 + R + AR')^2} \right](1 + R + AR'),$$

(6)

³ The dramatic increase in the rate of return associated with a shift from traditional to high yielding varieties (HYVs)—a shift that is contingent on investment in a bundled package of modern inputs—provides an excellent illustration. (See Stark 1978; this study also provides evidence that the proceeds of migrants who do earn high wages are used to undertake investment in projects with increasing returns.)
where \( U''(\cdot) \) and \( U'(\cdot) \) are evaluated at \( A[1 + R(A)] \). Thus locally, at \( W = C = 0 \), the isoultility curve is increasing, but its curvature is unknown. The expression in square brackets in (6) may be positive or negative, depending on \( A \) and the exact characteristics of the rate of return function \( R(A) \). The set of acceptable gambles, therefore, may or may not be convex. Thus the concavity of the utility function is neither necessary nor sufficient for the convexity of the acceptable gambles set. The conclusion, then, is that the individual may migrate and thus accept an unfair gamble even if his utility function is a concave function of wealth.

An examination of (6) shows that there are two effects to consider: (1) the effect of an increase in wealth on marginal utility, that is, the curvature of the utility function, and (2) the effect of an increase in wealth on the return to wealth, that is, the curvature of the wealth return function. The first effect, \( U''(\cdot)/U'(\cdot) \), is (minus) the Arrow-Pratt measure of (local) absolute risk aversion. In this case, however, we cannot look at this risk-aversion measure only; it is both effects that determine whether an individual with initial wealth, \( A \), undertakes an actuarially unfair migration gamble.

Hence, given a sufficiently large \( R' \), it is clear that the potential migrant may overcome his risk aversion and migrate to the urban area even if this involves an unfair gamble. This is because migration gives a person a chance of being able to reap for himself the high rewards associated, for example, with drastic modernization of farm production techniques. 4

Finally, it is important to note that most of the results are local in the sense that they correspond to a given level of initial wealth. As the level of wealth changes, the individual may modify his behavior even if his preferences remain the same. Hence, as the level of wealth increases, the likely decline in the effect of capital market constraints may reduce his apparent risk-loving behavior. This suggests that the rural rich will not migrate even if their preference function is no different from that of the rural poor.

Note that we have identified a new mechanism that accounts for rural-to-urban migration in LDCs: migration takes place because it enables a person to overcome a constraint imposed by the rural capital markets. Furnishing a person with even a small chance of reaping a large reward may make migration a game well worth the candle.

4 There is extensive evidence that migration income facilitates technological change in agricultural production (see Stark 1978) and is utilized to accumulate productive capital assets in the rural areas. For example, in Botswana migrants remit heavily and rely on their families back in the rural areas to act as trustworthy intermediaries in accumulating and maintaining capital—including human capital in the form of the education and upbringing of own children left behind in the rural areas (see Lucas and Stark 1985).
III

In this section we provide an alternative interpretation of the acceptance by migrants of an actuarially unfair gamble by migrating from rural to urban areas. Once again this is done without the need to relax the universal assumption of risk aversion. The explanation provided here is essentially an adaptation of that offered by Katz (1983) and Stark (1984a, 1984b).

It has recently become accepted that individuals derive utility from their wealth in two distinct ways. First, wealth yields benefits in terms of consumption. Second, wealth may provide a social status such that the greater the wealth of an individual in comparison with others, the greater his utility. To enable us to capture these two effects, let an individual's utility, $U$, be defined on his wealth, $W$, and on his social rank, $S$, such that $S$ depends on $W$; that is, let

$$U = U(W, S)$$

be the individual's utility function, and, to ensure risk aversion, assume that $U$ is strictly concave in $W$ and $S$. This implies that

$$U_1 > 0, \quad U_2 > 0, \quad U_{11} < 0, \quad U_{22} < 0$$

and

$$U_{11}U_{22} - U_{12}^2 > 0.$$ 

The social rank, $S$, may be measured as that proportion of the population (with which the migrant compares himself) that has an amount of wealth smaller than the individual's.\(^5\) Thus the lowest ranking is 0 and the top ranking is 1.

If $g(W)$ is the wealth density function for the relevant population, the social status of an individual with wealth $W$ is given by

$$S = \int_0^W g(W)dW = R(W).$$

If we now use our earlier assumptions about the parameters of the migration decision and ignore the complications generated by possible investment, we can show that the boundary of the set of acceptable gambles is defined by

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\(^5\) Such a comparison generates levels of relative satisfaction—or deprivation—that, in turn, can be modeled to impinge on migratory decisions. This direction is pursued in Stark (1984a) for the case of rural-to-urban migration and in Stark (1984b) in the context of international migration.
\[ U[A, S(A)] = qU[A + W, S(A + W)] + (1 - q)U[(A - C), S(A - C)]. \] (9)

Now, as mentioned earlier, the individual will not accept a fair gamble if and only if the boundary set is convex, that is, if the isouility curve defined by (9) is concave. Defining the isouility curve as

\[ C = G(W, A), \] (10)

it is proved in the Appendix that, locally, at \( W = C = 0, \)

\[ \frac{dC}{dW} = \frac{q}{1 - q} \] (11)

and

\[ \frac{d^2C}{dW^2} = \frac{q(U_1 + U_2S')^{-1}}{(1 - q)^2} [(U_{11} + 2U_{12}S' + U_{22}S'^2) + U_2S'']. \] (12)

Clearly, by concavity, the first term inside the square brackets, that is, \((U_{11} + 2U_{12}S' + U_{22}S'^2)\), is negative. However, for the concavity of \(G\), the concavity of \(U\) is neither necessary nor sufficient since it also depends on \(U_2S''\).

It transpires that \(S''\) may well be positive within low wealth ranges, given commonly accepted shapes of wealth distribution. For example, if the wealth distribution is normal, then the relation between \(S\) and \(W\) is as plotted in figure 1. Clearly, between \(W_1\) and \(W_2\), \(S''\) is positive, and if this effect is sufficiently strong, and if the marginal utility of status is sufficiently high, it may cause risk-averse individuals to engage in an actuarially unfair gamble through migration.

Holding the identity of the population with which the migrant compares himself as given, we obtain the same result as before but through an alternative route: a small prospect for a greatly enhanced status fueled by increased wealth—along with the increase in wealth itself—is sufficient to make migration preferable to nonmigration, even though the migrant is universally risk averse, both in rank and in wealth.

IV

We now offer some concluding comments. The assertion of the pioneering work of Todaro (1969) and Harris and Todaro (1970) that rural-to-urban migration in LDCs is an individual response to a higher urban expected income has been transformed by hundreds of articles and dozens of textbooks on development economics to an axiomatic postulate. In this paper we have questioned this status, demonstrating that rural-to-urban migration is rational even if urban expected income
is lower than the rural income. We achieve this under a set of fairly stringent conditions: an individual decision-making entity, a one-period planning horizon, and global risk aversion. And we obtain a quite powerful result: a small chance of reaping a high reward is sufficient to trigger rural-to-urban migration. Our result hinges on one of two explanations. The first is an explicit and, in our view, realistic assumption concerning the incompleteness of capital markets. Hence there is a cross-market spillover from cause to conduct. (Of course, in a general equilibrium context, disequilibrium in one market coincides with, and is causally related to, disequilibrium in another.) The policy implication suggested by this analysis is that manipulation of rural-to-urban migration of labor may require interference in capital markets. Improving the operational efficiency of financial markets, especially in rural areas, is very distinct from narrowing an intersectoral (expected) wage differential. At least in part, migration may be the intermediary between capital markets and labor markets where deficiency in the former is partially corrected. When we relate our analysis to the recent work on migration and relative deprivation, we present the second explanation, which demonstrates that migration may occur even if it constitutes an actuarially unfair risk. Individuals who care about both wealth (or income) and rank (relative deprivation or satisfaction) and who are globally risk averse in wealth and rank might still be better off taking actuarially unfair risks with respect to wealth in migrating to the urban area if by so doing they face even a small prospect of greatly enhancing their rank. The likelihood that migration will pay off depends crucially on the shape of the population wealth (or income) density function. Indeed, the strength of our results lies in the fact that, in general, wealth density functions are of the requisite shapes.

Our analysis leads us to identify a new research direction. We believe
that in real life people not only derive utility from rank and from absolute income but also use one to obtain more of the other. The underlying "production functions" may be such that rural-to-urban migration could render the transformation process of one into the other more productive. Thus, for example, migration may be used to improve rank in the rural-origin reference group by more than would have been possible by staying behind. If absolute income positively depends on relative position, migration advantageous with respect to the latter may subsequently facilitate improvement in the former. Given the interaction between rank and absolute income, the interesting effect of migration on the conversion of one into the other has been little explored. Perhaps particularly in LDCs, where, due to capital market imperfections, extension of credit by local money lenders may critically depend on rank (social status) as collateral, enhanced rank (status) captured through migration may translate quite powerfully into pecuniary gains.

Finally, it may be worthwhile to consider the difference between the testable implications of our explanations and the implications of Todaro’s approach. Let us begin with our imperfect capital markets explanation. There are two major differences between Todaro’s and our predictions. First, our model would predict that rural-to-urban migration will be mainly from rural areas characterized by a high marginal product of capital, coupled with a significant absence of capital markets. Todaro’s approach does not require such a bias. Put somewhat more bluntly, given the expected urban wage, Todaro’s hypothesis would predict least migration from rural areas of high production potential, while ours would seem to predict most migration from precisely such areas if and only if significant capital market imperfections existed there. Second, our model requires that the earnings of urban migrants are mainly channeled into investment activity. Todaro’s approach would predict a similar consumption to investment ratio both in the rural and in the urban areas.

As for our rank explanation, our model would predict migration by individuals whose rank is very sensitive to changes in income. Thus we would predict that rural-to-urban migrants are concentrated among rural individuals whose income’s position is in an upward-sloping portion of the income density function. Such a prediction would not be made by Todaro.

Appendix

I. Deriving the curvature of $C = G(W, A)$, as in (3), let

$$J = U\{(A + W)(1 + R(A + W))\} \quad \text{(A1)}$$

and

$$K = U\{(A - C)(1 + R(A - C))\}. \quad \text{(A2)}$$
Then, totally differentiating (3), we have

$$qJ_w dW + (1 - q)K_C dC = 0,$$

(A3)

so that

$$\frac{dC}{dW} = -\frac{qJ_w}{(1 - q)K_C}.$$  \hfill (A4)

Now

$$J_w = [(A + W)R'(A + W) + R(A + W) + 1]U'(A + W)[1 + R(A - C)]$$

(A5)

and

$$K_C = -[(A - C)R'(A - C) + R(A - C) + 1]U'(A - C)[1 + R(A - C)]$$

(A6)

Therefore, at $W = C = 0$,

$$J_w = -K_C = (AR' + R + 1)U'[A[1 + R(A)],$$

(A7)

and hence

$$\left. \frac{dC}{dW} \right|_{W=C=0} = -\frac{q}{1 - q}.$$  \hfill (A8)

From (A4),

$$\frac{d^2C}{dW^2} = -\frac{q}{1 - q} \left( K_C J_{ww} - K_{CC} J_w \frac{dC}{dW} \right)$$

(A9)

since $K_{CW} = J_{WC} = 0$.

Now from (A5),

$$J_{ww} = [(A + W)R'(A + W) + R(A + W) + 1]^2$$

$$\times U''(A + W)[1 + R(A + W)] + U'(A + W)[1 + R(A + W)]$$

$$\times [(A + W)R''(A + W) + 2R'(A + W)]$$

(A10)

and from (A6),

$$K_{CC} = [(A - C)R'(A - C) + R(A - C) + 1]^2$$

$$\times U''(A - C)[1 + R(A - C)] + U'(A - C)[1 + R(A - C)]$$

$$\times [(A - C)R''(A - C) + 2R'(A - C)]$$

(A11)
so that at $W = C = 0$,

$$J_{WW} = K_{CC} = (AR' + R + 1)^2 U'' + (AR'' + 2R')U'.$$

Hence, noting that

$$1 + \frac{dC}{dW} = \frac{1}{1 - q},$$

we know that

$$\frac{d^2C}{dW^2} = \frac{q}{(1 - q)^2} \left[ \frac{U''}{U'} + \frac{2R' + AR''}{(1 + R + AR')^2} \right] (1 + R + AR'),$$

which is equation (6).

II. Deriving the curvature of $C = G(W, A)$, as in (9), let

$$L = U[A + W, S(A + W)]$$

and

$$M = U[A - C, S(A - C)].$$

Then, totally differentiating (9), we have

$$qL W dW + (1 - q)M C dC = 0,$$

so that

$$\frac{dC}{dW} = -\frac{qL W}{(1 - q)M C}.$$

But

$$L W = U_1[A + W, S(A + W)] + U_2[A + W, S(A + W)]S'(A + W),$$

and

$$M C = -U_1[A - C, S(A - C)] - U_2[A - C, S(A - C)]S'(A - C),$$

so that at $W = C = 0$

$$L W = -M C = U_1 + U_2S',$$

and hence

$$\frac{dC}{dW} \bigg|_{W=C=0} = \frac{q}{1 - q}.$$
Hence, from (A18), we have

$$\frac{d^2C}{dW^2} = -\frac{q}{1 - q} \left( \frac{MC_{lw} - MC_{lw}}{M_C^2} \frac{d^2C}{dW} \right),$$  \tag{A23}

since $MC_{cw} = L_{wc} = 0$.

Hence, from (A19),

$$L_{ww} = U_{11}[A + W, S(A + W)] + U_{22}[A + W, S(A + W)]S'(A + W) + 2U_{12}[(A + W), S(A + W)]S' \tag{A24}$$

and from (A20),

$$MC_{cc} = U_{11}[A - C, S(A - C)] + U_{22}[A - C, S(A - C)]S'(A - C) + 2U_{12}[(A - C), S(A - C)]S' \tag{A25}$$

so that at $W = C = 0$

$$L_{ww} = MC_{cc} = U_{11} + U_{22}S^2 + 2U_{12}S' + U_2S'' \tag{A26}$$

Hence, using (A13), we know that

$$\frac{d^2C}{dW^2} = \frac{q}{(1 - q)^2} (U_1 + U_2S')^{-1}(U_{11} + U_{22}S^2 + 2U_{12}S' + U_2S''),$$  \tag{A27}

which is equation (12).

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